the flow in jets with close Mach numbers $M_a = 2.63$ and 2.78 and with different nozzle geometries shows that with an increase in the angle μ (see Fig. 1), i.e., with an increase in δ, the hysteresis phenomena decrease and disappear. Conversely, a decrease in the angle μ promotes the development of hysteresis. For example, for a jet with $M_a = 2.54$, $d/D = 0.753$, $\mu = 0$, and $\beta = 8°$ [4] (Fig. 6; points 1 correspond to an increase in n, and points 2 to a decrease in n) the hysteresis zone has the maximum range of expansion ratios. Thus, hysteresis phenomena in supersonic jets essentially depend on the Mach number and the profiling of the nozzles.

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AERODYNAMIC FORCES ACTING ON THE BLADES OF A THREE-DIMENSIONAL ANNULAR ARRAY WITH NONSTEADY FLOW

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UDC 532.5:621.22

We present a computer realization of the solution of the three-dimensional problem of the nonsteady flow over the vane crown of an axial turbine by an irrotational stream of an ideal incompressible fluid, based on the vortex theory of a screw [1] and of a wing of finite span [2].

To solve this problem in [3, 4] the geometry of a blade crown was modeled by a straight three-dimensional array of plates enclosed between two planes, while in [5, 6] it was modeled by an annular array of vanes consisting of parts of helical surfaces. In the present report we adopt the second model, which evidently better describes the geometry of an actual turbine.

Because of the complexity of the algorithm suggested in [4–6], there are only individual examples of the calculation of nonsteady aerodynamic characteristics. Below, on the basis of a simple algorithm which is a generalization of the working method of [7] for an established flow, we analyze the influence of the three-dimensionality of the flow on the nonsteady aerodynamic forces acting on the vanes of a round array in a wide range of variation of the parameters of the array.

1. Let us consider a uniform stream of an ideal incompressible fluid with an axial velocity v through one array of vanes which are rotating with a constant angular velocity ω in a coaxial cylindrical channel which is infinite in the axial direction. We assume that the vanes can undergo synchronous, steady, harmonic vibrations of low amplitude at a frequency ω_1 and a constant phase shift $\mu\pi$ ($\mu = 2\sigma/N$, where $\sigma = 0, \pm1, \pm2, \ldots$; N is the number of vanes in the array).

We introduce cartesian (x, y, z) and cylindrical (x, r^*, θ^*) coordinate systems connected with the rotating vane array. The x axis is directed along the axis of rotation while the y and z axes are drawn in the plane perpendicular to it. The r^* and θ^* coordinates are connected with y and z by the usual equations, $y = r^* \cos\theta^*$ and $z = r^* \sin\theta^*$, where the angle θ^* is reckoned in the positive direction from the y axis (Fig. 1).

We assume that the vanes Σ_n $(n = 0, \ldots, N-1)$ are infinitely thin and in the central position they consist of parts of helical surfaces bounded in the (r^*, θ^*) plane by a rectangle $\{r_1 \le r^* \le r_2,\ \alpha_n - \psi \le \theta^* \le \alpha_n + \psi\}$. Here $\alpha_n = 2\pi n/N$; n is the number of vanes; r_1 and r_2 are the radii of the inner and outer cylin-

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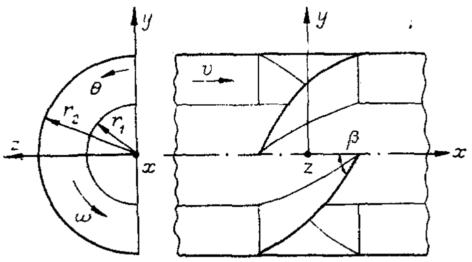

Fig. 1

ders, respectively; $\alpha_n - \psi$ and $\alpha_n + \psi$ are angles defining the positions of the leading and trailing edges of the n-th vane. Within the framework of the linear theory the vortex wakes developing behind vanes owing to the variation of the circulation with time and over the height of a vane will be modeled by surfaces W_n of discontinuity of the tangential velocities, distributed along the stream surfaces of the undisturbed flow and bounded in the (r^*, θ^*) plane by the half-sheet $\{r_1 \le r^* \le r_2, \alpha_n + \psi \le \theta^* < \infty\}$. The helical surfaces Σ_n and the free vortical surfaces W_n $(n = 0, 1, \ldots, N-1)$ will be defined by the equations

$$x = r_1 m\theta,\ y = r_1 r\cos(\theta + \alpha_n),\ z = r_1 r\sin(\theta + \alpha_n), \tag{1.1}$$

where $m = v/\omega r_1$; $r = r^*/r_1$ is the dimensionless radial coordinate; the angle $\theta = \theta^* - \alpha_n$.

We will assume that the disturbed nonsteady motion of the fluid outside the vanes and the vorti cal wakes is potential. Then the following problem arises for the determination of the potential φ of the disturbed velocities:

$$\begin{cases} \Delta\varphi = 0 \text{ outside } \Sigma_n \text{ and } W_n \quad (n = 0, 1, \ldots, N-1), \\ (\mathbf{v}\cdot\nabla)\varphi = V_\nu(\mathbf{x})\, e^{j(n\mu\pi + \omega_1 t)},\ \mathbf{x} \in \Sigma_n, \\ [p] = 0,\ [(\mathbf{v}\cdot\nabla)\varphi] = 0 \quad \text{for } \mathbf{x} \in W_n,\ \lim\limits_{x\to-\infty} \nabla\varphi = 0, \\ \dfrac{\partial\varphi}{\partial r} = 0 \text{ for } r = 1 \text{ and } h,\ |\nabla\varphi| < \infty \quad \text{for } \mathbf{x} \in L_n, \end{cases} \tag{1.2}$$

where $\boldsymbol{\nu}$ is the vector normal to the surfaces Σ_n and W_n; V_ν is the normal component of the amplitude function of the oscillation velocity of the first vane $(n = 0)$; p is the pressure, determined by the Cauchy–Lagrange integral; L_n is the line of the trailing edge of the n-th vane; $\mathbf{x} = (x, y, z)$; t is the time; j is an imaginary unit connected only with transient processes; $h = r_2/r_1$; brackets denote a jump in the quantity which they contain.

2. To solve the problem (1.2) we replace the vanes of the array by vortex surfaces and change from a continuous to a discrete distribution of the vortices, by analogy with how this is done in [7] in calculating steady flow over a vane crown. We divide a vane into N_1 bands in r and N_2 bands in θ and model each i-th elementary area by a horseshoe-shaped vortex consisting of a segment of the attached vortex directed along the r axis and having a size $2\delta r = (h-1)/N_1$ and with an intensity

$$\Gamma_{+i}^{(n)}(t) = r_1 v_0 \Gamma_i e^{j(\omega_1 t + n\mu\pi)}, \tag{2.1}$$

and a system of free vortices. Here Γ_i is a dimensionless constant which is complex with respect to j; v_0 is the velocity of the undisturbed stream at the middle radius $r = r_0 = (h+1)/2$ of the channel.

According to the Kelvin theorem, the variation in the intensity of the attached vortices is accompanied by the coming off of the free vortices with an intensity

$$\gamma_-(s_i, t) = -\frac{1}{v_1(r_i)} \left.\frac{d\Gamma_{+i}^{(n)}}{dt}\right|_{t=t_i}. \tag{2.2}$$

Here $t_i = t - s_i/v_1(r_i)$, $v_1(r) = \sqrt{v^2 + \omega^2 r^{*2}}$, and $s_i = r_1\sqrt{r_i^2 + m^2}\,\theta$ is the curved coordinate of the streamline of the undisturbed flow, measured from the point (r_i, θ_i), where r_i is the coordinate of the middle of the segment of the i-th attached vortex; θ_i is its angular coordinate. Substituting (2.1) into (2.2), we obtain

$$\gamma_-(s_i, t) = -\frac{m\Gamma_i v_0}{\sqrt{m^2 + r_i^2}}\, jq e^{j(\omega_1 t + n\mu\pi - qm\theta)},$$

where $q = \omega_1 r_1/v$ is the Strouhal number. The axes of these vortices are parallel to the axes of the attached vortices. The system of free vortices also includes two semiinfinite vortex strings coming off the ends of the attached vortex and located on the helical lines defined by Eqs. (1.1) with $r = r_i \pm \delta r$. At the time t the intensities of these vortex strings are $\Gamma_{+i}^{(n)}(t_1)$ in magnitude and opposite in sign.

The coordinates of the attached vortices and of the control points, at which one determines the velocity induced by the vortex system, are determined in the (r, θ) plane just as in [7]. If s is the number of the band in θ $(s = 1, \ldots, N_2)$, σ is the number of the band in r $(\sigma = 1, \ldots, N_1)$, and l is the number of the horseshoe-shaped vortex, then we can introduce the numbering

$$l = N_1(s - 1) + \sigma$$

and determine the coordinates of the middles of the segments (r_l, θ_l) of the attached vortices and the control points (r_{0l}, θ_{0l}) from the equations

$$r_{0l} = r_l = h + \delta r(1 - 2\sigma), \ \theta_l = \psi((0{,}5 + 2(s - 1))/N_2 - 1),$$
$$\theta_{0l} = \psi((1{,}5 + 2(s - 1))/N_2 - 1).$$

Using the Biot–Savart law and the equation for determining the normal to the surface of a vane at the point (r_{0l}, θ_{0l}),

$$\mathbf{v} = \frac{\mathbf{v}_1}{\sqrt{m^2 + r_{0l}^2}}, \quad \mathbf{v}_1 = (r_{0l}, m \sin\theta_{0l}, -m\cos\theta_{0l}).$$

for the normal velocity component at this point of the i-th attached vortex of the n-th vane we obtain

$$v_{\mathrm{v}-}^{i}(r_{0i}, \theta_{0l}) = -\frac{v_0\Gamma_i e^{j(\omega_1 t + n\mu\pi)}}{4\pi\sqrt{m^2 + r_{0l}^2}} a_{il}^{n}(\vartheta_{il}) \left[\frac{r_i + \delta r - r_{0l}\cos(\vartheta_{il} - \alpha_n)}{R(r_i + \delta r, \vartheta_{il})} - \frac{r_i - \delta r - r_{0l}\cos(\vartheta_{il} - \alpha_n)}{R(r_i - \delta r, \vartheta_{il})}\right], \tag{2.3}$$

where

$$a_{il}^{n}(\theta) = \frac{r_{0l}^2 \sin(\theta - \alpha_n) + m^2\theta\cos(\theta - \alpha_n)}{m^2\theta^2 + r_{0l}^2 \sin^2(\theta - \alpha_n)};$$

$$R^2(r, \theta) = m^2\theta^2 + r_{0l}^2 + r^2 - 2rr_{0l}\cos(\theta - \alpha_n); \ \vartheta_{il} = \theta_{0l} - \theta_i.$$

For the normal component of the velocity induced by the free vortices coming off the i-th attached vortex we have

$$v_{\mathrm{v}1}^{i}(r_{0l}, \theta_{0l}) = \frac{jqmv_0\Gamma_i e^{j(\omega_1 t + n\mu\pi - qm\vartheta_{il})}}{4\pi\sqrt{m^2 + r_{0l}^2}} \int_{-\infty}^{\vartheta_{il}} e^{jqmx} a_{il}^{n}(x) \left[\frac{r_i + \delta r - r_{0l}\cos(x - \alpha_n)}{R(r_i + \delta r, x)} - \frac{r_i - \delta r - r_{0l}\cos(x - \alpha_n)}{R(r_i - \delta r, x)}\right] dx. \tag{2.4}$$

Similarly, for the free vortex string belonging to the i-th horseshoe-shaped vortex and having the coordinate r along the height of a vane, we obtain

$$v_{\mathrm{v}-}^{i}(r_{0l}, \theta_{0l}, r) = -\frac{v_0\Gamma_i}{4\pi\sqrt{m^2 + r_{0l}^2}} e^{j(\omega_1 t + n\mu\pi - qm\vartheta_{il})} \int_{-\infty}^{\vartheta_{il}} e^{jqmx} \frac{r_{0l}(r^2 - m^2) + r^2(m^2 - r_{0l}^2)\cos(x - \alpha_n) + rm^2 x \sin(x - \alpha_n)}{R^3(r, x)} dx. \tag{2.5}$$

Then the normal component of the velocity induced by the i-th horsehoe-shaped vortex at the point (r_{0l}, θ_{0l}) has the form

$$v_{\mathrm{v}}^{i}(r_{0l}, \theta_{0l}) = v_{\mathrm{v}-}^{i}(r_{0l}, \theta_{0l}) + v_{\mathrm{v}1}^{i}(r_{0l}, \theta_{0l}) + v_{\mathrm{v}-}^{i}(r_{0l}, \theta_{0l}, r_i + \delta r) - v_{\mathrm{v}-}^{i}(r_{0l}, \theta_{0l}, r_i - \delta r) = w_{li}^{n}\Gamma_i.$$

However, the conditions of nonpenetration at the cylinder surfaces $r = 1$ and h cannot be satisfied using the vortex system under consideration. To approximately satisfy them we introduce, following [7], a supplementary vortex system which is a reflection relative to the cylinders $r = 1$ and h of the vortex system of the vanes in each cross section $x = \text{const}$. A method permitting an exact allowance for the boundary conditions at the cylinders was also suggested in [7] for the solution of the problem of steady flow over an annular array with $h \approx 1$. This method is based on the use of Fourier integrals and modified Bessel functions. For an array with a density $\tau = 1$, an entrainment angle $\beta = 30°$, and a vane number $N = 4$ the coefficient of the total force acting on a vane is $C_{n\alpha} = 2.2$ for $h = 2$. In calculations by the first method for $h = 20$, 10, 5, and 2 we obtained $C_{n\alpha} = 1.7533$, 1.7393, 1.7238, and 1.7073, respectively. In this case the angle of attack varied linearly over the height of a vane from $\alpha(1) = 0.15$ to $\alpha(h) = 0.05$. The decrease in the coefficient of force in the given case can be explained by the fact that, in contrast to the method for $h \approx 1$, the variability of the load in the radial direction is taken into account. It should be noted that with a considerable change in h the coefficient of force changes little and is close to the value obtained when the boundary conditions at the cylinders are exactly satisfied. In addition, a direct calculation of the radial velocities at the cylinders from all the vortex systems was made in the analysis of the method. These quantities did not exceed 0.01 in the examples considered. One can therefore expect that the method using reflected vortex systems also gives satisfactory results in a solution of the nonsteady problem.

The normal components of the velocities induced by the reflected vortex system are determined from Eqs. (2.3)–(2.5), in which one must replace $r_i \pm \delta r$ by $1/(r_i \pm \delta r)$ or by $h^2/(r_i + \delta r)$, respectively.

Now requiring that the condition of nonpenetration of the surface of the first vane be satisfied, we obtain a system of algebraic equations, complex with respect to j, for determining the intensity of the attached vortices,

$$AX = B, \tag{2.6}$$

where A is a matrix with the elements

$$A_{li} = \sum_{n=0}^{N-1} (w_{li}^n - w_{1li}^n - w_{2li}^n),$$

$i, l = 1,\dots,M$, $M = N_1 \times N_2$, the quantities w_{1li}^n and w_{2li}^n determine the normal velocities produced by the reflected system; $X = \{\Gamma_i\}$ is the vector constructed from the unknown intensities of the attached vortices; $B = \{V_\nu(r_{0l}, \theta_{0l})\}$.

Finding Γ_i from the system (2.6), we can calculate the nonsteady aerodynamic characteristics both for an entire vane and for sections of it in height. Defining the pressure drop at a vane from the Zhukovskii theorem "in small,"

$$[p] = -\rho v_1(r)\gamma_+ \ (r, \theta),$$

and an element of vane area from the equation

$$dS = r_1^2 \sqrt{m^2 + r^2}\, dr d\theta,$$

we find that the aerodynamic force acting on a vane is

$$P = -\rho r_1^2 \int_1^h \int_{-\psi}^{\psi} v_1(r) \sqrt{m^2 + r^2}\, \gamma_+(r, \theta)\, dr d\theta \tag{2.7}$$

or, in dimensionless form,

$$C_n = \operatorname{Re} C_n + j \operatorname{Im} C_n = P \Big/ \frac{1}{2} \rho v_0^2 S. \tag{2.8}$$

Here $\gamma_+(r, \theta)$ is the intensity of the attached vortices continuously distributed over the vane surface; ρ is the fluid density.

Substituting (2.7) into (2.8), we replace $\gamma_+ \sqrt{m^2 + r_i^2}\, d\theta$ by $v_0 \Gamma_i$, dr by $2\delta r$, and change from integrals to finite sums. Then

$$C_n = -\frac{4\delta r}{S_1 \sqrt{m^2 + r_0^2}} \sum_{i=1}^{M} \sqrt{m^2 + r_i^2}\, \Gamma_i.$$

Similarly, for the coefficient of aerodynamic force acting on the l-th cross section ($l = 1,\dots,N_1$) over the height of a vane, we have

$$C_{nl} = -\frac{4\delta r \sqrt{m^2 + r_0^2}}{S_{1l}(m^2 + r_l^2)} \sum_{i=1}^{N_2} \sqrt{m^2 + r_i^2}\, \Gamma_{N_1 i + l}.$$

where $C_{nl} = P_l/(0.5)\rho v_1^2(r_l) S_l$; $S_1 = S/r_1^2$ and $S_{1l} = S_l/r_1^2$ are the dimensionless area of a vane and of its l-th band in height, respectively.

The algorithm for calculating the improper integrals entering into Eqs. (2.4) and (2.5) has considerable influence on the accuracy and time of the calculation. In constructing it we factor the integrands into two parts such that one of them contains almost all the singularities and its integral is found exactly, while the integral of the second part can be calculated by mechanical quadratures with a small error. As the first part it is convenient to take integrands with $n = 0$ and $x \ll 1$, since the singularities when the control point (r_{0l}, θ_{0l}) lies near a horseshoe-shaped vortex are eliminated in this case. We represent the resulting integrals in the form

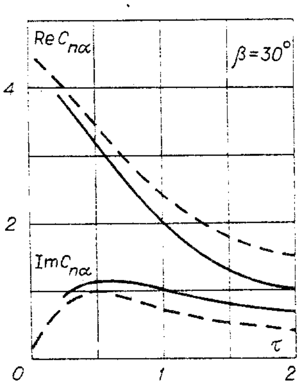

Fig. 2

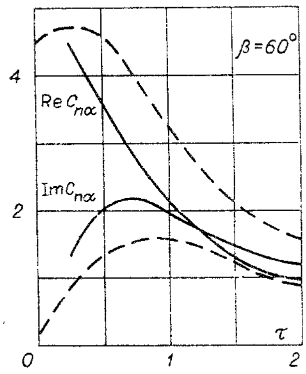

Fig. 3

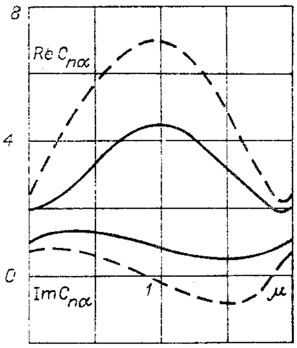

Fig. 4

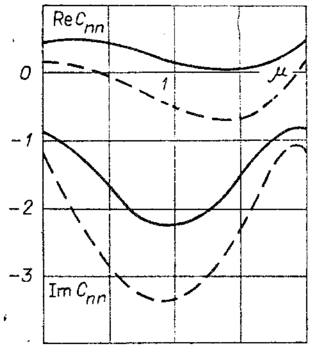

Fig. 5

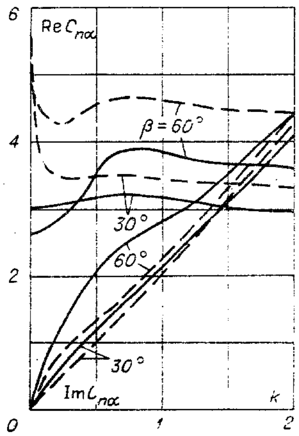

Fig. 6

$$\int_{-\infty}^{\vartheta_{il}} f(x)\,dx = \int_{-\Delta}^{\vartheta_{il}} f(x)\,dx + \int_{-\infty}^{-\Delta} f(x)\,dx, \tag{2.9}$$

where f(x) is an integrand function without a singularity, while $\Delta \gg 1$ is chosen in such a way that the second integral can be neglected while making only a small error. We use the following procedure to calculate the first integral on the right side of (2.9). We consider the indefinite integral

$$y(\theta) = \int_{-\Delta}^{\theta} f(x)\,dx$$

for values of θ assigned in the interval $[-\Delta, 2\psi]$. We calculate the values of the function y for a given grid of values of the argument θ: $\theta_n = -\Delta + nh_1$ $(n = 0, 1, \ldots)$, from the equation $y_{n+1} = y_n + 0.5h_1(f_n + f_{n+1})$ for a trapezoid. Then using linear interpolation, we find the required values of the integrals at $\theta = \vartheta_{il}$. The proposed method lets us entirely eliminate repeated calculations of the integrand and thereby considerably reduce the computer time needed to calculate the elements of the matrix A.

3. The distributed and total nonsteady aerodynamic characteristics of a number of annular arrays were calculated on a BÉSM-6 computer by the algorithm presented. Since unrolling a cylindrical cross section of the annular arrays under consideration onto a plane gives an array of plates, we compared the results obtained with the data of [4]. In this case the parameters characterizing the geometry of the array and the vibration process were determined at the middle radius of the annular channel: $m = r_0 \cot \beta$ (β is the entrainment angle of the array), the density of the array is $\tau = \psi\sqrt{m^2 + r_0^2}N/(\pi r_0)$, the vane aspect ratio is $\lambda = (h-1)^2/S_1$, and the Strouhal number $k = \omega_1 b_0/v_0 = 2r_1\omega_1\psi\sqrt{m^2 + r_0^2}/v_0$ is connected with the quantity q by the relation $k = 2\psi\sqrt{m^2 + r_0^2}\,q$. Here b_0 is the length of the chord of a vane at the middle radius of the channel.

Below we consider bending–twisting vibrations of a vane as a rigid body in the cross section r = const. In the case of twisting vibrations relative to some center with the coordinate $\theta = \theta_0$ and an angle $\alpha(t) = \varepsilon e^{j\omega_1 t}$ the right side of the system of equations (2.6) has the form

$$V_\nu(r_{0l}, \theta_{0l}) = \varepsilon \sqrt{\frac{m^2 + r_l^2}{m^2 + r_0^2}}\, v_0 \left[jqm(\theta_{0l} - \theta_0) + 1\right],$$

where $\varepsilon \ll 1$ is the angular amplitude of the vane vibrations.

To determine the coefficient of the nonsteady aerodynamic force acting on an entire vane or a cross section of it in height during bending vibrations we made calculations for two twisting centers and used the equation

$$C_{n\alpha} = C_{n\alpha 0} - \frac{|\theta_0|}{2\psi} C_{nn},$$

where $C_{n\alpha_0} = C_{n0}/\alpha(t)$ is the coefficient of the normal force during twisting vibrations relative to the middle of a vane ($\theta_0 = 0$); C_{nn} is the coefficient of force during bending vibrations in the direction of the normal to the vane surface.

The calculations presented below were made with h = 2. In Figs. 2 and 3 we present the coefficients of nonsteady aerodynamic forces $C_{n\alpha} = \text{Re } C_{n\alpha} + f \text{ Im } C_{n\alpha}$ during twisting vibrations about the middle of a vane as functions of the density τ of the array for entrainment angles $\beta = 30$ and 60°, respectively. Here the Strouhal number is $k = 0.5$, the phase shift is $\mu\pi = 0$, and the number of vanes is $N = 4$. In Figs. 2–6 the dependences for the force acting on the middle cross section of the vane are shown by solid lines while those for the force calculated for the corresponding array of plates [4] are shown by dashed lines. As seen from these calculations, the difference between the results is slight for $\beta = 30°$; for larger entrainment angles ($\beta = 60°$) this difference becomes important. Thus, one can conclude that the effects of the three-dimensionality of the flow are manifested to a greater degree with an increase in the entrainment angle.

The coefficients of nonsteady aerodynamic forces as functions of the phase shift $\mu\pi$ between the vibrations of neighboring vanes during twisting vibrations about the middle and bending vibrations are shown in Figs. 4 and 5, respectively. The calculations were made for values of $\tau = 1, \beta = 30°, k = 0.5, N = 4$, and $\mu = \sigma/2$ ($\sigma = 0, 1, 2, 3$). A comparison with the results of the plane theory shows that the maximum difference is observed for vibrations in antiphase ($\mu = 1$). This fact also follows from the calculation results presented in [5].

The nonsteady aerodynamic coefficients as functions of the Strouhal number k at $\beta = 30$ and 60° are shown in Fig. 6. Here we took $\tau = 0.5, \mu = 0, \theta_0 = 0$, and $N = 4$. It should be noted for $\beta = 30°$ the results of calculations by the proposed method and for an array of plates are also very close in this case, except for the region of $k = 0$ where we did not observe a sharp change in the aerodynamic characteristics as in the plane theory [4]. For $\beta = 60°$ the difference in the results is considerable, especially in the region of small Strouhal numbers ($k \approx 0.5$), while the influence of the three-dimensionality of the flow decreases with an increase in the Strouhal number.

The values of the coefficients of the total force and of the force in the middle cross section practically coincide in these examples.

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NONSTEADY ESCAPE OF GAS INTO A VACUUM THROUGH A SEMIPERMEABLE SCREEN

G. I. Gannochenko

UDC 533.6.011.533.697

The problem of the distribution of the parameters of a gas in a rarefaction wave during the nonsteady escape of the gas into a vacuum through a screen, which has a hydrodynamic resistance and removes part of the gas energy, is solved by the method of the theory of similarity and dimensionalities.

Suppose that a plane $x = 0$ (Fig. 1) separates a left-hand half-space $x < 0$, filled with an ideal gas having the parameters ρ_0, p_0, and T_0 and an equation of state $p = \rho T$, from a right-hand half-space, a vacuum 1 ($x > 0$).

At some moment the gas starts to escape into the vacuum through an infinitely thin screen 2 located in this plane which possesses a hydrodynamic resistance and removes part of the energy of the stream. The front of a rarefaction wave 3 propagates away from the screen to the left (through the undisturbed gas) and the boundary of the expanding gas 4 propagates to the right. The parameters of the flow in the rarefaction wave to the left of the plane $x = 0$ have the index 1 while the parameters of the flow to the right of this plane have the index 2.

We assume that the specific flow rate of gas through the screen depends on the pressure drop at the screen in the following way:

$$q = \alpha(p_{10} - p_{20}),$$

where p_{10} and p_{20} are the gas pressures at the plane $x = 0$ to the left and right of the screen; α is the coefficient of permeability of the screen.

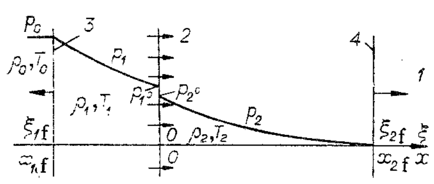

Fig. 1

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 97–102, July–August, 1979. Original article submitted July 3, 1978.

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